Gauss Stylization: Interactive Artistic Mesh Modeling based on Preferred Surface Normals

Maximilian Kohlbrenner, Ugo Finnendahl, Tobias Djuren, Marc Alexa

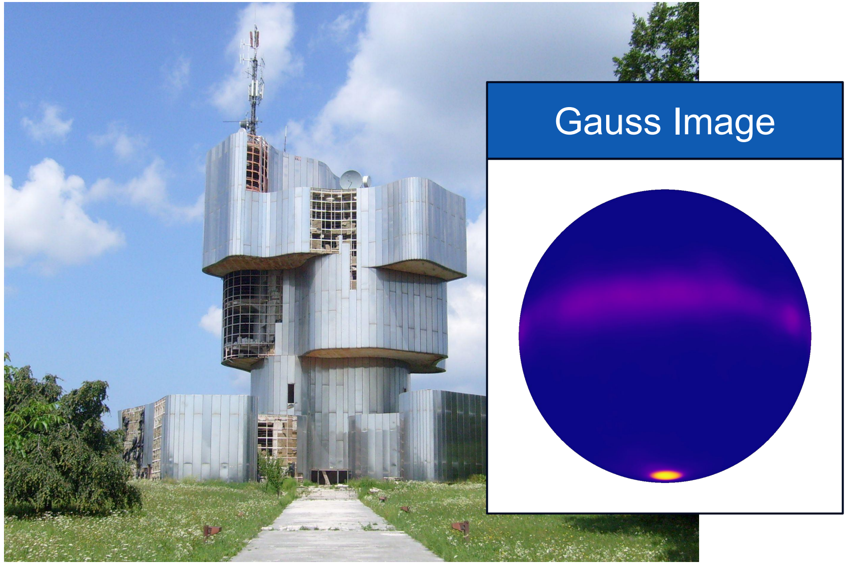
TU Berlin

Artistic Stylization

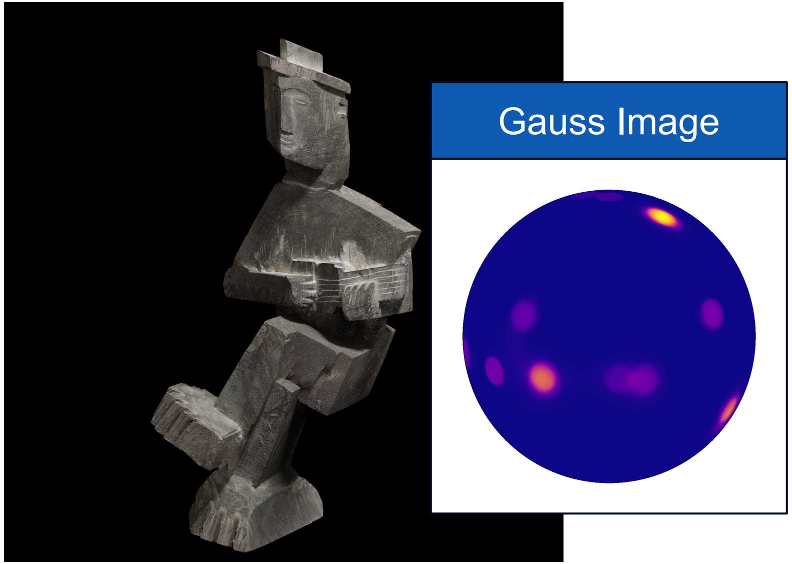
© Picture by Sandor Bordas under CC BY-SA.

Original model © Sculpture Man with Guitar by Världskulturmuseerna under CC BY

Artistic Stylization

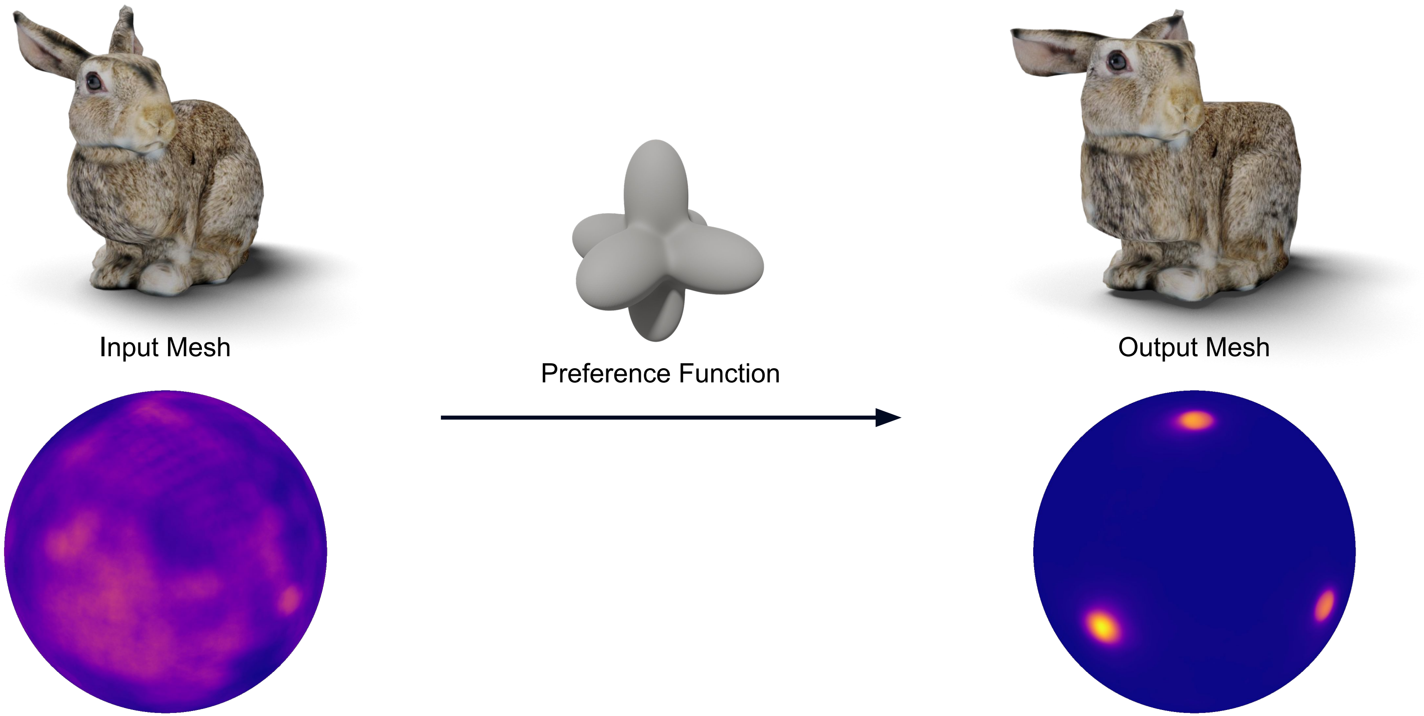


© Picture by Sandor Bordas under CC BY-SA.



Original model © Sculpture Man with Guitar by Världskulturmuseerna under CC BY

Artistic Style and the Gauss Image



© Bunny texture by yellokab under CC BY

Placeholder: Interactive Modelling Session and Rotating Dinosaur

Gauss Stylization

Cubic Stylization

Hsueh-Ti Derek Liu and Alec Jacobson. 2019. Cubic stylization. *ACM Trans. Graph.* 38, 6, Article 197 (November 2019), 10 pages.
DOI:https://doi.org/10.1145/3355089.3356495

Cubic Stylization

$$E_{\square}(\mathbf{V},\{\mathbf{R}\}) = \sum_{i\in\mathcal{V}} \sum_{j\in N(i)} \frac{w_{ij}}{2} \|\mathbf{e}_{ij} - \mathbf{R}_i \hat{\mathbf{e}}_{ij}\|^2 + \lambda \sum_{i\in\mathcal{V}} a_i ||\mathbf{R}_i \hat{\mathbf{n}}_i||_1$$

Graphic:
Hsueh-Ti Derek Liu and Alec Jacobson. 2019. Cubic stylization. *ACM Trans. Graph*. 38, 6, Article 197 (November 2019), 10 pages. DOI:https://doi.org/10.1145/3355089.3356495

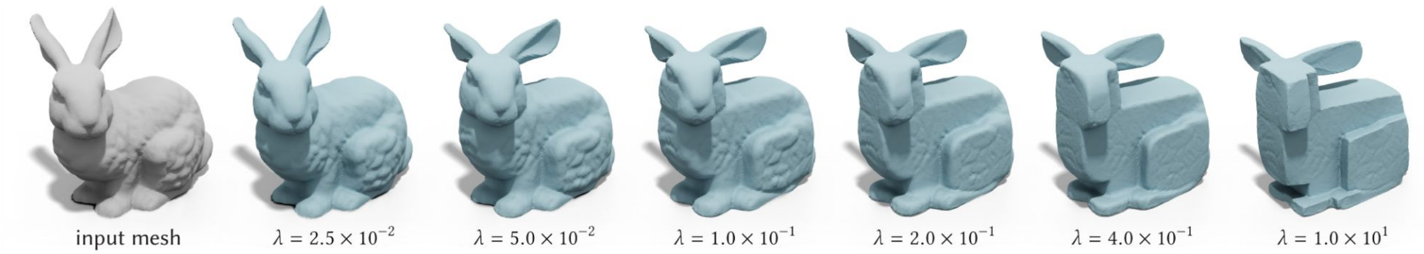
Cubic Stylization

$$E_{\square}(\mathbf{V}, \{\mathbf{R}\}) = \boxed{\sum_{i \in \mathcal{V}} \sum_{j \in N(i)} \frac{w_{ij}}{2} \|\mathbf{e}_{ij} - \mathbf{R}_i \hat{\mathbf{e}}_{ij}\|^2} + \lambda \sum_{i \in \mathcal{V}} a_i \|\mathbf{R}_i \hat{\mathbf{n}}_i\|_1$$

As-Rigid-As-Possible (ARAP) energy:
Preserve local neighborhoods

Graphic:
Hsueh-Ti Derek Liu and Alec Jacobson. 2019. Cubic stylization. *ACM Trans. Graph.* 38, 6, Article 197 (November 2019), 10 pages.
DOI:https://doi.org/10.1145/3355089.3356495

Cubic Stylization



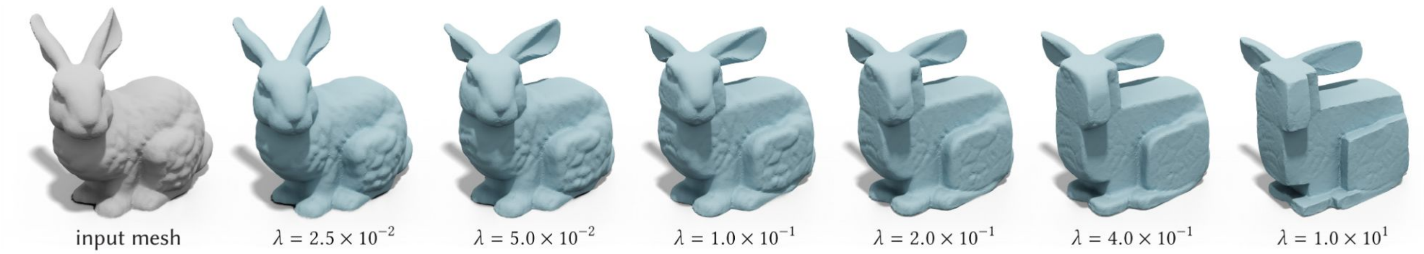
$$E_{\square}(\mathbf{V},\{\mathbf{R}\}) = \sum_{i\in\mathcal{V}} \sum_{j\in N(i)} \frac{w_{ij}}{2} \|\mathbf{e}_{ij} - \mathbf{R}_i \hat{\mathbf{e}}_{ij}\|^2 + \lambda \sum_{i\in\mathcal{V}} a_i \|\mathbf{R}_i \hat{\mathbf{n}}_i\|_1$$

As-Rigid-As-Possible (ARAP) energy:
Preserve local neighborhoods

Cubeness term:
Align rotated vertex normals with the coordinate axes

Graphic:
Hsueh-Ti Derek Liu and Alec Jacobson. 2019. Cubic stylization. *ACM Trans. Graph.* 38, 6, Article 197 (November 2019), 10 pages. DOI:https://doi.org/10.1145/3355089.3356495

Cubic Stylization



Lambda parameter: weighting

$$E_{\square}(\mathbf{V},\{\mathbf{R}\}) = \sum_{i\in\mathcal{V}}\sum_{j\in N(i)} \frac{w_{ij}}{2}\|\mathbf{e}_{ij}-\mathbf{R}_i\hat{\mathbf{e}}_{ij}\|^2 + \lambda\sum_{i\in\mathcal{V}} a_i\|\mathbf{R}_i\hat{\mathbf{n}}_i\|_1$$

As-Rigid-As-Possible (ARAP) energy:
Preserve local neighborhoods

Cubeness term:
Align rotated vertex normals with the coordinate axes

Graphic:
Hsueh-Ti Derek Liu and Alec Jacobson. 2019. Cubic stylization. *ACM Trans. Graph.* 38, 6, Article 197 (November 2019), 10 pages. DOI:https://doi.org/10.1145/3355089.3356495

Gauss Stylization

$$E_G(\mathbf{V}, \{\mathbf{R}_i\}) = E_{\mathrm{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f)$$

Gauss Stylization

Mu parameter: weighting

$$E_G(\mathbf{V}, \{\mathbf{R}_i\}) = E_{\text{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f)$$

As-Rigid-As-Possible (ARAP) energy:
Preserve local neighborhoods

Stylization term

Gauss Stylization

Mu parameter: weighting

$$E_G(\mathbf{V}, \{\mathbf{R}_i\}) = E_{\text{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f)$$

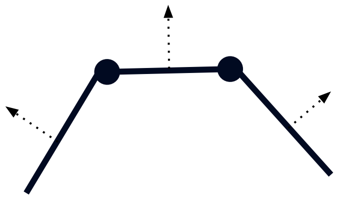
As-Rigid-As-Possible (ARAP) energy:
Preserve local neighborhoods

Stylization term:
- Face normals
- g function

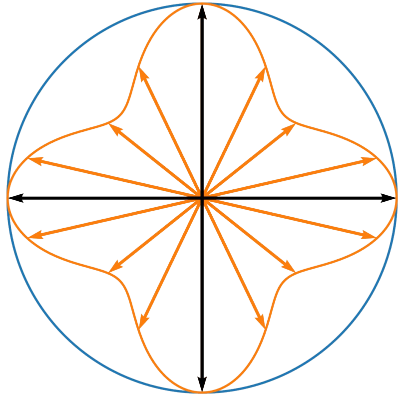
Gauss Stylization

$$E_G(\mathbf{V}, \{\mathbf{R}_i\}) = E_{\mathrm{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f)$$

(1) Face-wise Approach



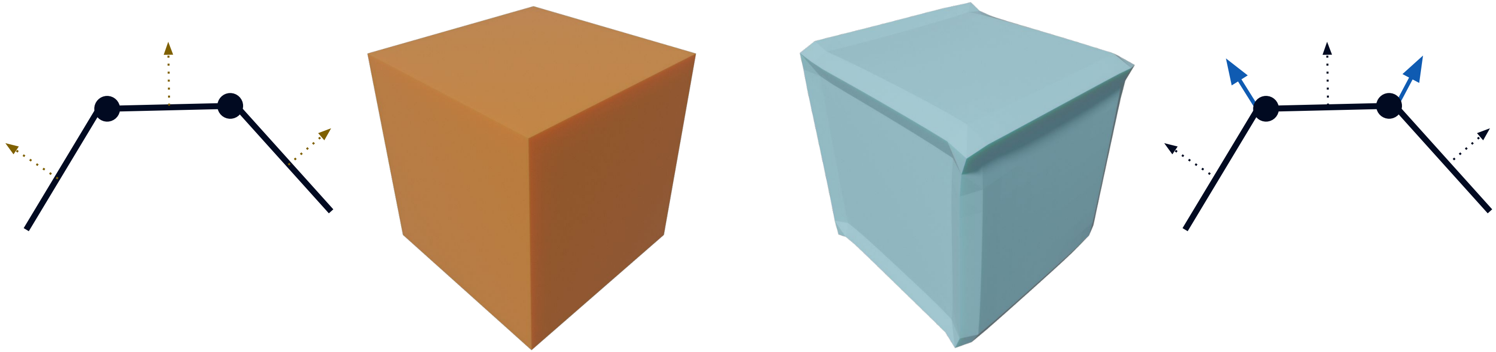
(2) Generalized Preference Function



Gauss Stylization

$$E_G(\mathbf{V}, \{\mathbf{R}_i\}) = E_{\text{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f)$$

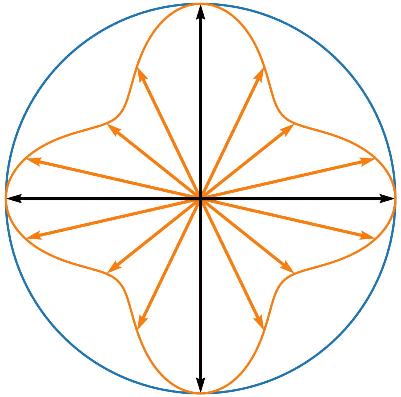
(1) Face-wise Approach



Gauss Stylization

$$E_G(\mathbf{V}, \{\mathbf{R}_i\}) = E_{\text{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f)$$

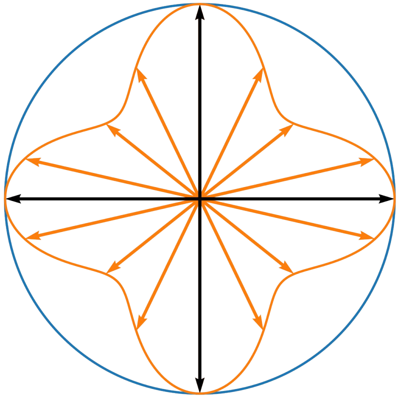
(2) Generalized Preference Function

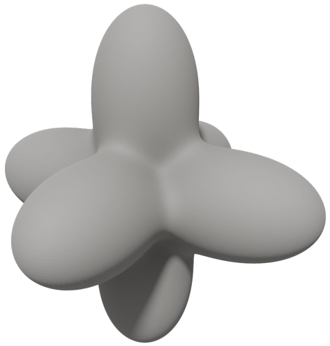


Gauss Stylization: g Function

$$E_G(\mathbf{V}, \{\mathbf{R}_i\}) = E_{\text{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f)$$

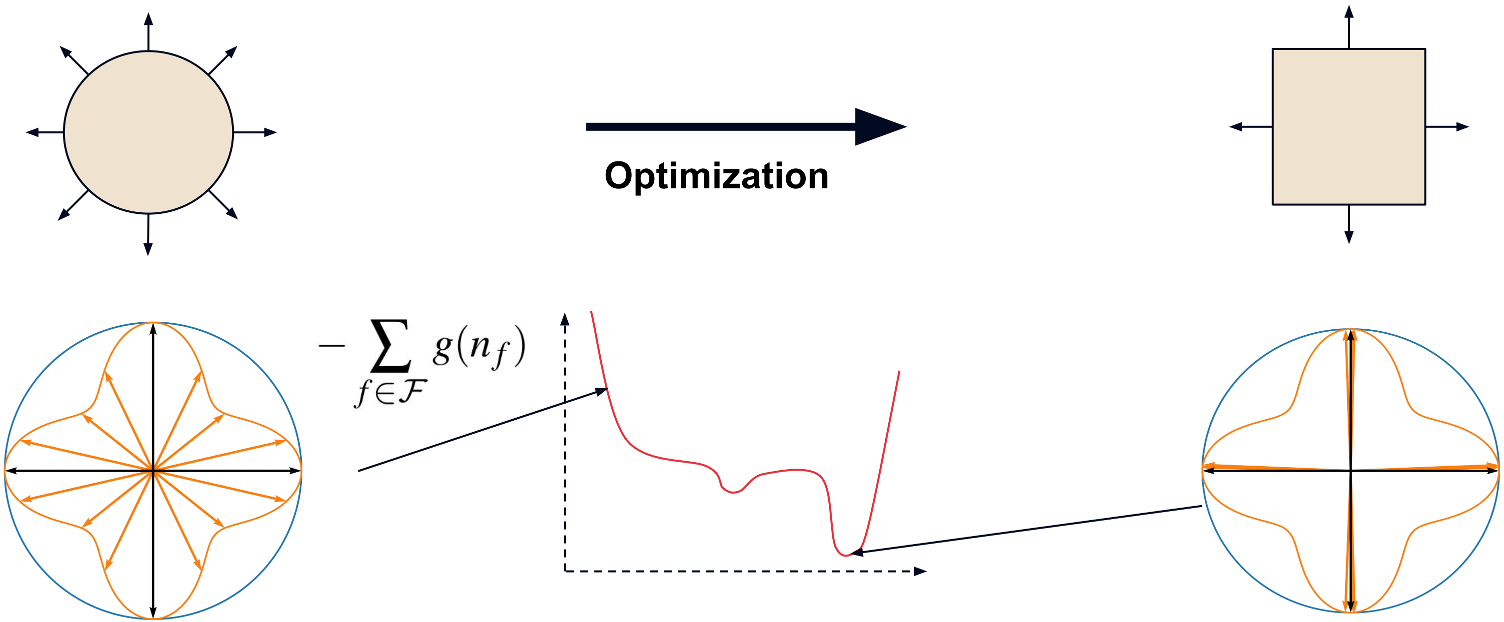
g function
• Function defined on surface normals (unit length) • No fixed general form



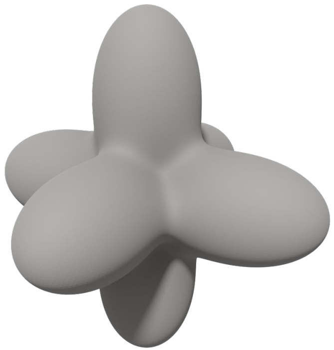


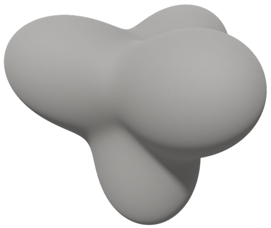
Cube g function

Normal Stylization and the Gauss Map

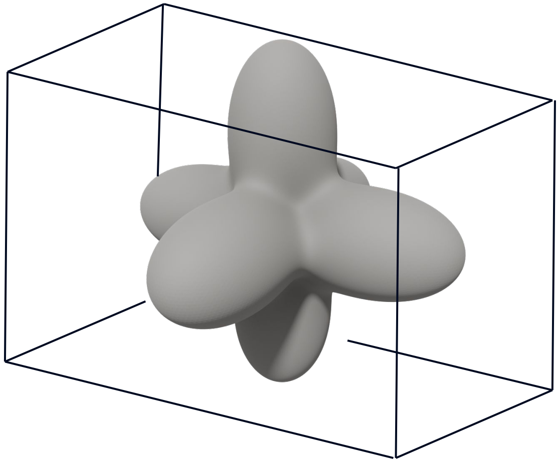


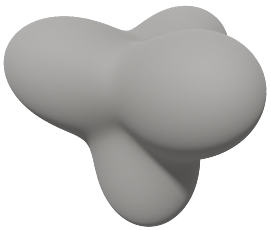
Normal Stylization and the Gauss Map



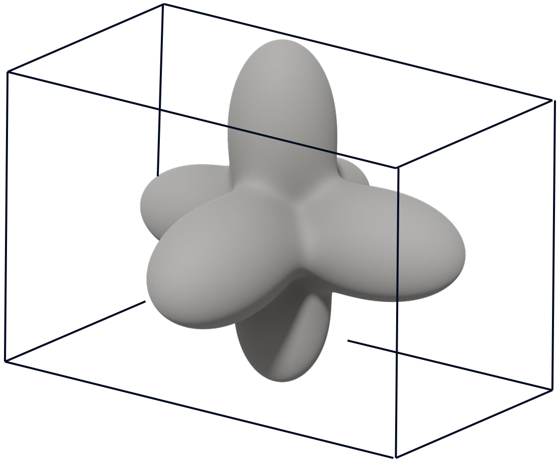


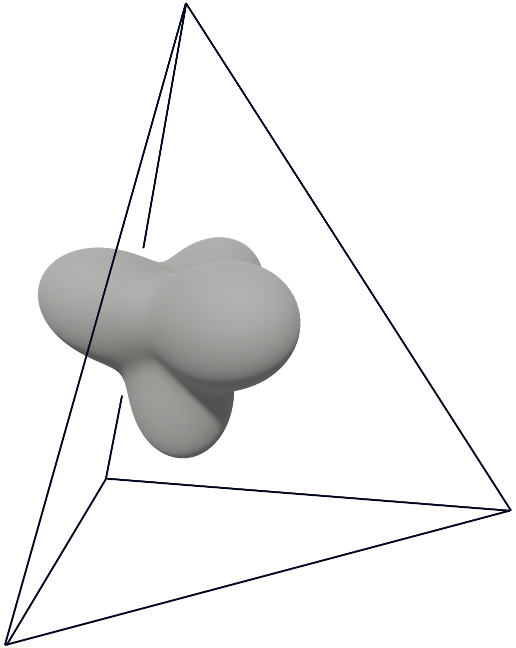
Normal Stylization and the Gauss Map





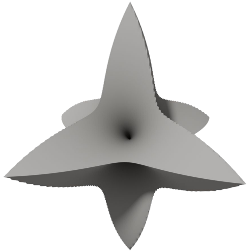
Normal Stylization and the Gauss Map





L1 Based Cubification

Cubic Stylization



L2 Based Cubification

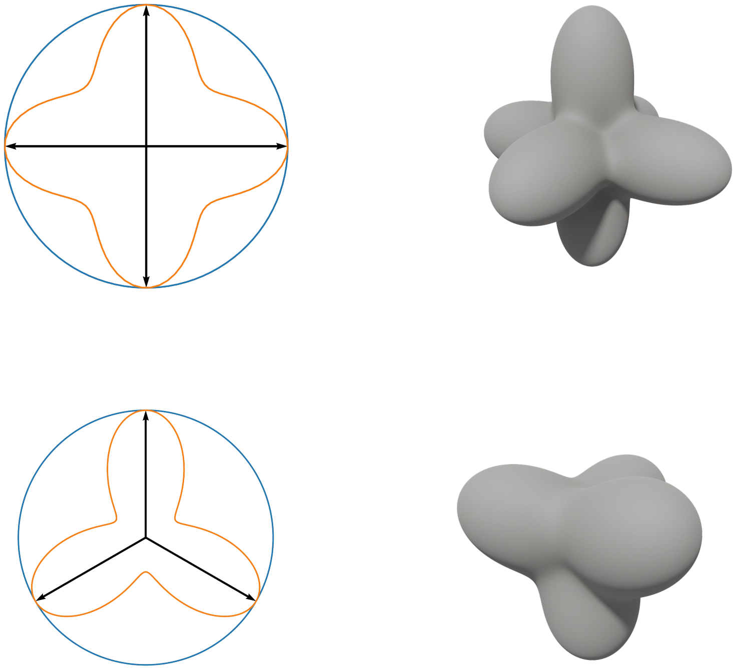
Cubic Stylization

Gauss Stylization
(L2 preference function)

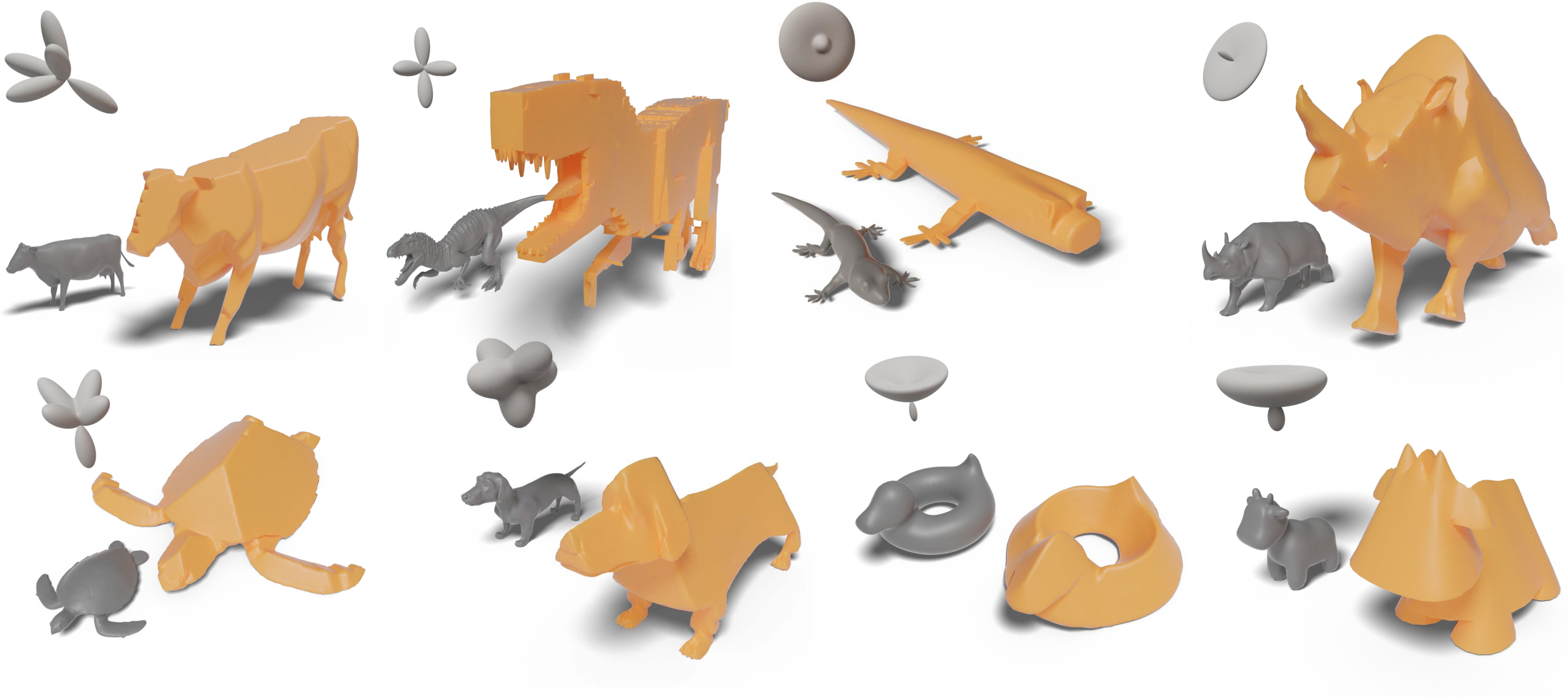
Modelling the g Function

$$g(\mathbf{x}) = \sum_k w_k \exp(\boldsymbol{\sigma} \mathbf{x}^\top \mathbf{n}_k)$$

- Intuitive modelling
- Fast calculation of function value, gradient and hessian (optimization)



Placeholder: modelling sessions



Copyrights of the original models from left to right and top to bottom: © Cow by Josué Boisvert, © Indominus Rex dinosaur by BlueMesh, © Tokay Gecko by DigitalLife3, © Rhino by Gremory Saiyan, © Hawksbill Turtle by Bindestrek and © Dog by Pusztai Andras under CC BY.

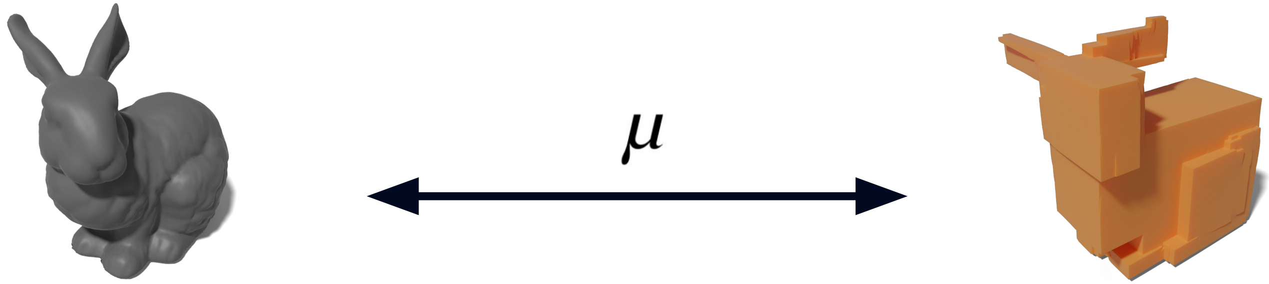
Optimization

Energy Formulation

Stylize Surface Normals

$$E_G(\mathbf{V},\{\mathbf{R}_i\}) = E_{\text{ARAP}}(\mathbf{V},\{\mathbf{R}_i\}) - \mu \sum_{f\in\mathcal{F}} g(\mathbf{n}_f)$$

Preserve Local Geometry



Note: ARAP Optimization

$$E_G(\mathbf{V}, \{\mathbf{R}_i\}) = E_{\text{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f)$$

ARAP Optimization
Alternating Block Descent: ● Local Step (R) Polar Decomposition ● Global Step (V) system of linear equations (SLE) with constant system matrix

Energy Formulation

$$E_G(\mathbf{V}, \{\mathbf{R}_i\}) = E_{\text{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f)$$

Energy Formulation

Non-Convex in V

$$E_G(\mathbf{V}, \{\mathbf{R}_i\}) = E_{\text{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f)$$

Properties:

- Non-Convex

Energy Formulation

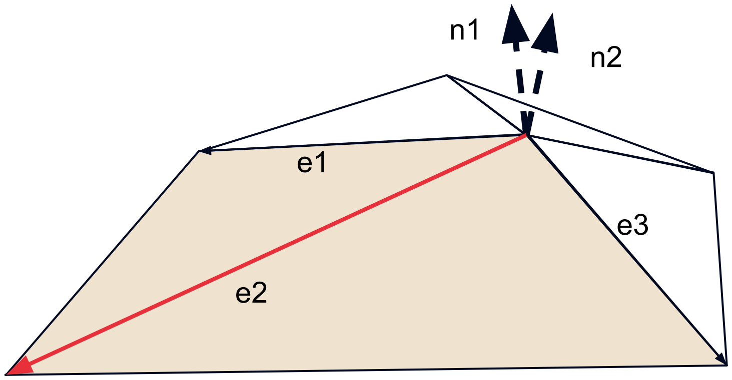
Non-Convex in V

$$E_G(\mathbf{V}, \{\mathbf{R}_i\}) = E_{\text{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f)$$

Properties:

- Non-Convex
- Global

Global Step:
g term influences system matrix



Energy Formulation

$$E_G(\mathbf{V}, \{\mathbf{R}_i\}) = E_{\mathrm{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f)$$

Properties:

- Non-Convex
- Global

We propose: Approximation

- Allows for local/global block descent split
- Constant system matrix

Decoupling The Normals

$$E_G^*(\mathbf{V}, \mathbf{R}, \mathbf{n}^*, \mathbf{e}^*) = E_{\text{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f^*)$$

$$+ \lambda \sum_{(i,j) \in \mathcal{E}} \frac{w_{ij}}{2} \|\mathbf{e}_{ij} - \mathbf{e}_{ij}^*\|^2,$$

$$\text{s.t. } \mathbf{n}_f^{*\mathsf{T}} \mathbf{e}_{ij}^* = 0, \quad (i, j) \in f$$

Decoupling The Normals

$$E_G^*(\mathbf{V}, \mathbf{R}, \mathbf{n}^*, \mathbf{e}^*) = E_{\text{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f^*)$$

$$+ \lambda \sum_{(i,j) \in \mathcal{E}} \frac{w_{ij}}{2} \|\mathbf{e}_{ij} - \mathbf{e}_{ij}^*\|^2,$$

$$\text{s.t. } \mathbf{n}_f^{*\top} \mathbf{e}_{ij}^* = 0, \quad (i,j) \in f$$

Decoupling The Normals

$$E_G^*(\mathbf{V},\mathbf{R},\mathbf{n}^*,\mathbf{e}^*) = E_{\text{ARAP}}(\mathbf{V},\{\mathbf{R}_i\}) - \mu \sum_{f\in\mathcal{F}} g(\mathbf{n}_f^*)$$

$$+\lambda \sum_{(i,j)\in\mathcal{E}} \frac{w_{ij}}{2}\|\mathbf{e}_{ij}-\mathbf{e}_{ij}^*\|^2,$$

$$\text{s.t. } \mathbf{n}_f^{*\mathsf{T}} \mathbf{e}_{ij}^* = 0, \quad (i,j)\in f$$

Decoupling The Normals

$$E_G^*(\mathbf{V}, \mathbf{R}, \mathbf{n}^*, \mathbf{e}^*) = E_{\text{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f^*)$$

$$+ \lambda \sum_{(i,j) \in \mathcal{E}} \frac{w_{ij}}{2} \|\mathbf{e}_{ij} - \mathbf{e}_{ij}^*\|^2,$$

$$\text{s.t. } \mathbf{n}_f^{*\mathsf{T}} \mathbf{e}_{ij}^* = 0, \quad (i,j) \in f$$

Decoupling The Normals

$$E_G^*(\mathbf{V},\mathbf{R},\mathbf{n}^*,\mathbf{e}^*) = E_{\text{ARAP}}(\mathbf{V},\{\mathbf{R}_i\}) - \mu \sum_{f\in\mathcal{F}} g(\mathbf{n}_f^*)$$

$$+\lambda \sum_{(i,j)\in\mathcal{E}} \frac{w_{ij}}{2} \|\mathbf{e}_{ij} - \mathbf{e}_{ij}^*\|^2,$$

$$\text{s.t. } \mathbf{n}_f^{*\,\mathsf{T}} \mathbf{e}_{ij}^* = 0, \quad (i,j)\in f$$

Block Descent
3 Sets of Variables • V • R • n*, e*

Decoupling The Normals

$$E_G^*(\mathbf{V}, \mathbf{R}, \mathbf{n}^*, \mathbf{e}^*) = E_{\text{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f^*)$$

$$+ \lambda \sum_{(i,j) \in \mathcal{E}} \frac{w_{ij}}{2} \|\mathbf{e}_{ij} - \mathbf{e}_{ij}^*\|^2,$$

$$\text{s.t. } \mathbf{n}_f^{*\top} \mathbf{e}_{ij}^* = 0, \quad (i,j) \in f$$

Local Rotations R

- Same as in ARAP
- Polar Decomposition
- Local

Decoupling The Normals

$$E_G^*(\mathbf{V}, \mathbf{R}, \mathbf{n}^*, \mathbf{e}^*) = E_{\mathrm{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f^*)$$

$$+ \lambda \sum_{(i,j) \in \mathcal{E}} \frac{w_{ij}}{2} \|\mathbf{e}_{ij} - \mathbf{e}_{ij}^*\|^2,$$

$$\text{s.t. } \mathbf{n}_f^{*\top} \mathbf{e}_{ij}^* = 0, \quad (i,j) \in f$$

Vertex Positions V
Leads to SLE with constant system matrix

Decoupling The Normals

$$\sum_{i \in \mathcal{V}} \sum_{j \in N(i)} \frac{w_{ij}}{2} \| \mathbf{e}_{ij} - \mathbf{R}_i \hat{\mathbf{e}}_{ij} \|^2$$

$$E_G^*(\mathbf{V}, \mathbf{R}, \mathbf{n}^*, \mathbf{e}^*) = E_{\text{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f^*)$$

$$+ \lambda \sum_{(i,j) \in \mathcal{E}} \frac{w_{ij}}{2} \| \mathbf{e}_{ij} - \mathbf{e}_{ij}^* \|^2,$$

$$\text{s.t. } \mathbf{n}_f^{*\top} \mathbf{e}_{ij}^* = 0, \quad (i,j) \in f$$

Vertex Positions V
Leads to SLE with constant system matrix

Decoupling The Normals

$$E_G^*(\mathbf{V}, \mathbf{R}, \mathbf{n}^*, \mathbf{e}^*) = E_{\text{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f^*)$$

$$+ \lambda \sum_{(i,j) \in \mathcal{E}} \frac{w_{ij}}{2} \|\mathbf{e}_{ij} - \mathbf{e}_{ij}^*\|^2,$$

$$\text{s.t. } \mathbf{n}_f^{*\mathrm{T}} \mathbf{e}_{ij}^* = 0, \quad (i,j) \in f$$

Auxiliary n*, e*

- Indep. Terms
- Coupled by bi-convex constraint
- ADMM (iterative)

Decoupling The Normals

$$E_G^*(\mathbf{V}, \mathbf{R}, \mathbf{n}^*, \mathbf{e}^*) = E_{\mathrm{ARAP}}(\mathbf{V}, \{\mathbf{R}_i\}) - \mu \sum_{f \in \mathcal{F}} g(\mathbf{n}_f^*)$$

$$+ \lambda \sum_{(i,j) \in \mathcal{E}} \frac{w_{ij}}{2} \|\mathbf{e}_{ij} - \mathbf{e}_{ij}^*\|^2,$$

$$\text{s.t. } \mathbf{n}_f^{*\top} \mathbf{e}_{ij}^* = 0, \quad (i,j) \in f$$

Auxiliary n*, e*
• Indep. Terms • Coupled by bi-convex constraint • ADMM (iterative)

Algorithm 1: Gauss Stylization Update

Input : original vertex positions $\hat{V}$
current vertex positions V_n
ADMM iterations N

Output: updated vertex positions V_{n+1}

for $f \in F$ **do** *// Initialization*
  $\mathbf{n}_f^* \leftarrow \mathbf{n}_f$
  for $\mathbf{e}_{ij} \in E_f$ **do**
    $\mathbf{e}_{ij}^* \leftarrow \mathbf{e}_{ij}, u_{fij} \leftarrow 0$

for $i \in 1..N$ **do** *// ADMM optimization*
  for $f \in F$ **do**
    $\Delta\mathbf{n}_f^* \leftarrow \text{newton}(g, \mathbf{n}_f^*, u_{fij})$ (Eq. 21)
    $\mathbf{n}_f^* \leftarrow \text{normalize}(\mathbf{n}_f^* + (\Delta\mathbf{n}_f^* - \Delta\mathbf{n}_f^{*\top}\mathbf{n}_f^* \cdot \mathbf{n}_f^*))$
  for $\mathbf{e}_{ij} \in E$ **do**
    $\mathbf{e}_{ij}^* \leftarrow \text{lssolve}(\mathbf{n}_f^*, \mathbf{e}_{ij}^*, u_{fij})$ (Eq, 24)
  for $f \in F$ **do**
    for $\mathbf{e}_{ij} \in E_f$ **do**
      $u_{fij} \leftarrow u_{fij} + \mathbf{e}_{ij}^{*\top}\mathbf{n}_f^*$

for $v \in \hat{V}$ **do** *// local ARAP step*
  $R_v \leftarrow \text{local}(\hat{V}, V_n)$

$V_{n+1} \leftarrow \text{global}(V_n, \{R_i\}, \{\mathbf{e}_{ij}^*\})$ (Eq. 27) *// global step*

$\mathbf{n}^*, \mathbf{e}^*$ — Newton Step, Local LES

$\mathbf{R}$ — Polar decomposition

$\mathbf{V}$ — LSE, constant system matrix

Algorithm 1: Gauss Stylization Update

Input : original vertex positions $\hat{V}$
current vertex positions V_n
ADMM iterations N

Output: updated vertex positions V_{n+1}

for $f \in F$ **do** *// Initialization*
- $\mathbf{n}_f^* \leftarrow \mathbf{n}_f$
- **for** $\mathbf{e}_{ij} \in E_f$ **do**
  - $\mathbf{e}_{ij}^* \leftarrow \mathbf{e}_{ij}, u_{fij} \leftarrow 0$

for $i \in 1..N$ **do** *// ADMM optimization*
- **for** $f \in F$ **do**
  - $\Delta\mathbf{n}_f^* \leftarrow \text{newton}(g, \mathbf{n}_f^*, u_{fij})$ (Eq. 21)
  - $\mathbf{n}_f^* \leftarrow \text{normalize}(\mathbf{n}_f^* + (\Delta\mathbf{n}_f^* - \Delta\mathbf{n}_f^{*\top}\mathbf{n}_f^* \cdot \mathbf{n}_f^*))$
- **for** $\mathbf{e}_{ij} \in E$ **do**
  - $\mathbf{e}_{ij}^* \leftarrow \text{lssolve}(\mathbf{n}_f^*, \mathbf{e}_{ij}^*, u_{fij})$ (Eq, 24)
- **for** $f \in F$ **do**
  - **for** $\mathbf{e}_{ij} \in E_f$ **do**
    - $u_{fij} \leftarrow u_{fij} + \mathbf{e}_{ij}^{*\top}\mathbf{n}_f^*$

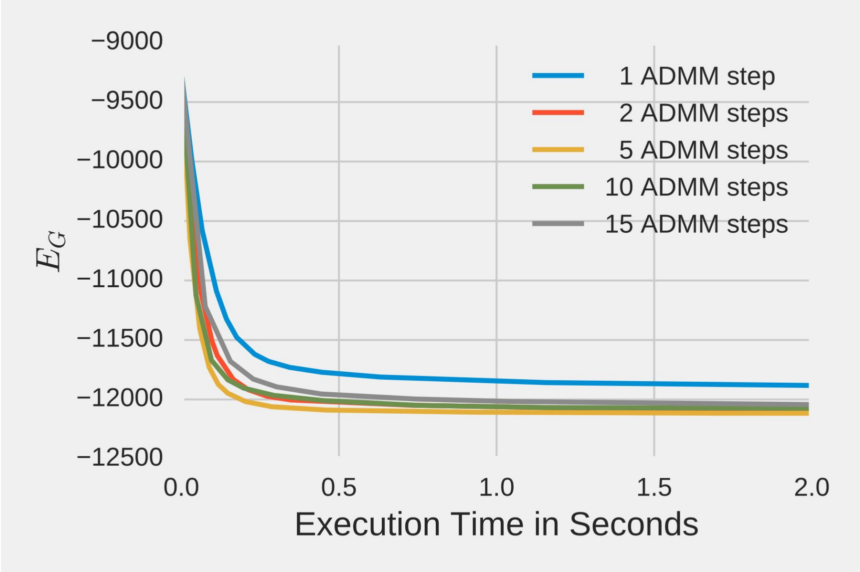
for $v \in \hat{V}$ **do** *// local ARAP step*
- $R_v \leftarrow \text{local}(\hat{V}, V_n)$

$V_{n+1} \leftarrow \text{global}(V_n, \{R_i\}, \{\mathbf{e}_{ij}^*\})$ (Eq. 27) *// global step*

for $f \in F$ **do**
- $\Delta\mathbf{n}_f^* \leftarrow \text{newton}(g, \mathbf{n}_f^*, u_{fij})$ (Eq. 21)
- $\mathbf{n}_f^* \leftarrow \text{normalize}(\mathbf{n}_f^* + (\Delta\mathbf{n}_f^* - \Delta\mathbf{n}_f^{*\top}\mathbf{n}_f^* \cdot \mathbf{n}_f^*))$

Energy Optimization

- Original energy effectively decreases
- In practice: 1 ADMM step for increased visual feedback



Good moment to insert a nice animation

http://cybertron.cg.tu-berlin.de/projects/gaussStylization/

Try It Yourself!

Original © Shiba Dog by zixisun02 under CC BY

Thank you for your attention!